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THE COSMOLOGICAL INFLATION

The Inflationary paradigm (Guth 1981) refers to the period in the very early universe (10^{-34}second) during which the scale factor of the universe evolved exponentially.

This solves at once several problems in the standard Hot Big Bang Model:
(i) The Flatness problem: why is the curvature of the universe so close to zero today?
(ii) The Horizon problem: why two regions on opposite sides of the sky have the same temperature (to within one part in 10^5) ?
(iii) How to dilute monopoles and other defects?
(iv) What is the origin of density fluctuations?

1 A simple derivation of the exponential expansion

The key idea is that at a certain early epoch the universe was dominated by the energy density of the vacuum $\rho_v \approx$ constant (possibly similar in nature but much larger today's cosmological constant).
Consider now an equation of state for this vacuum energy of the form:

$$p_v = -\rho_v c^2$$

Then from the fluid equation:

$$\dot{\rho_v} + 3\frac{\dot{a}}{a}\left(\rho_v + \frac{p_v}{c^2}\right) = 0,$$

we see immediately that $\rho_v =$ constant.
Recall now the Friedmann equation, with the curvature k and the Cosmological Constant Λ:

$$H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}.$$

If $\rho = \rho_v$ dominates relative the radiation, matter, curvature and the Λ term then we are just left with:

$$H_v^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_v.$$

This can easily be solved to give:

$$a(t) \propto \exp(H_v t).$$

Inflation can work to solve the above four problems if typically $H_v t \approx 65$. The key point is that a small causally connected region can grow to a much larger scale at present.
Consider now the acceleration equation:

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3}.$$

We can easily see that the above equation of state will give rise to an accelerating expansion ($\ddot{a} > 0$) during the inflationary period.
The inflation phase must come to an end so the standard Big Bang can continue. Most probably the vacuum energy decays into matter.

2 Inflation and early universe physics

(LJS version)
The universe at the very earliest stages of its evolution went through an exponentially rapid expansion while it was in a type of unstable vacuum-like state. This vacuum-like state is often called a false vacuum and has the the property that it has a high energy density that cannot be rapidly lowered. The driving force of inflation is the negative pressure or repulsive gravitational field.

An inflationary model therefore starts with a small patch in the early Universe which somehow came to be in a false vacuum state. A false vacuum naturally arises in theories of particle physics aimed at unifying the forces because to accomplish this, scalar fields (fields with no direction) are required.

For example, the mediators of the electromagnetic and weak forces are the massless photon and the heavy W and Z particles. To unify these two forces, a scalar field is required. The closest analogy to this is the electrostatic potential – if the whole universe has the same electrostatic potential, the field would not be apparent – it would just be another vacuum state. The main difference between the constant electrostatic potential and the scalar field is that it does not have its own energy unlike the scalar field which may have a potential energy density $V(\phi)$. If $V(\phi)$ has one minimum at $\phi = \phi_0$, then the whole universe eventually becomes filled by the field ϕ_0. This field is invisible, but, if it interacts with the W and Z particles, they become heavy. Meanwhile, if photons do not interact with the scalar field, they remain massless.

Inflation models start at a time in which all particles are light and there is no fundamental difference between the weak and electromagnetic interactions. The difference appears later when the universe becomes filled with the scalar field ϕ. At this moment, the symmetry between different types of fundamental interactions becomes broken.
The state in which the energy density of the scalar field is above the minimum ϕ_0 at the top of the plateau corresponds to the false vacuum. If the plateau is flat enough, it can take a very long time (by early Universe standards) for the scalar field to "roll" down the hill so that the energy density can be lowered. The flatter the plateau, the greater the amount of inflation – these models are called "slow roll-over" inflationary models.

Eventually the false vacuum decays, and the energy it contains is released. This energy produces a hot, uniform soup of particles which corresponds to the starting point of the traditional Hot Big Bang model, and all its successes.

The Universe starts incredibly small $\approx 10^{-26}$ m (10^{11} times smaller than the size of a proton). Through inflation, our Universe can start from very little – just a few ounces of primordial matter.

For inflation to be a true theory, there has to be an explanation of why it occurs. It is thought that a false vacuum state may arise due to a "phase transition". An analogy is water freezing. Very near the beginning, all four forces are unified and the Universe undergoes successive phase transitions as each force separates or freezes out. At each transition, its properties will change dramatically. One transition that inflationary modellers have focused on is the one where the strong force obtains an identity distinct from the electro-weak force which freezes out later. This occurs at 10^{16} GeV or 10^{-34} s.

Another (more likely) possibility is super symmetry – a theory of particle physics which postulates that every fundamental particle we know about (photon, electron, quark) has a partner particle with higher mass. In the early Universe, they would have had very similar properties and then a phase transition would lead to their present separate identities. Their higher mass means they are impossible to detect using particle accelerators so they are entirely theoretical at present, or they might not exist at all!