# SCM Lecture 10 - teaching notes

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## 1 Weighted Average

What if we have a number of measurements of the same quantity, $X$, but with different precision. The best estimate of $X$ is not the mean - it should be closer to the more precise measurement, so when we calculate our average the more precise measurement should count more.
Use the weighted mean.

Set of measurements $x_i \pm \sigma_i$, we give each measurement the weight $w_i = \frac{1}{\sigma_i^2}$. The best measurement for the mean is

$$\bar{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} = \frac{\sum_{i=1}^{n} \frac{x_i}{\sigma_i^2}}{\sum_{i=1}^{n} \frac{1}{\sigma_i^2}} \tag{1}$$

and the error on this mean value is

$$\sigma_{\bar{x}} = \sqrt{\frac{1}{\sum_{i=1}^{n} w_i}} \tag{2}$$

If the errors are equal, these formulas simplify to the basic mean and standard deviation formulas.

*Proof*
Take two measurements $x_A \pm \sigma_A$ and $x_B \pm \sigma_B$. The chi-squared of these is:

$$\chi^2 = \frac{(x_A - \bar{x})^2}{\sigma_A^2} + \frac{(x_B - \bar{x})^2}{\sigma_B^2} \tag{3}$$

where $\bar{x}$ is the average we are trying to find. To find the best estimator of $x$ we need to minimise the chi-squared, so differentiate and set it to zero.

$$\frac{d\chi^2}{dx} = 2\frac{x_A - \bar{x}}{\sigma_A^2} + 2\frac{x_B - \bar{x}}{\sigma_B^2} = 0 \tag{4}$$

solve for $\bar{x}$:

$$\bar{x} = \frac{\left(\frac{x_A}{\sigma_A^2} + \frac{x_B}{\sigma_B^2}\right)}{\left(\frac{1}{\sigma_A^2} + \frac{1}{\sigma_B^2}\right)} \tag{5}$$

And extending that to the case of more than two variables brings us back to the stated formula.