3. Ferromagnetic Materials (etc.)

3.1 Description of Ferromagnetism at the Atomic Level

A number of neutral atoms, in the region of the periodic table close to iron, and elsewhere, have incompletely filled inner shells, with an unpaired electron spin. This gives the atom a residual magnetic moment which defines an axis vector. As always in quantum mechanics, once a direction is chosen then other properties of the atom (e.g. the electron configurations in the outermost shells) can be aligned along the same axis. In solids there is a strong short range force between neighbouring atoms due to the interaction of their outermost shells. They will choose the configuration which gives the lowest potential energy under the action of this force. In ferromagnetic materials this turns out to be with nearest neighbour axes parallel to one another. In antiferromagnetic materials the nearest neighbour axes are opposite to one another. There are subtle variations; see drawing from Ashcroft and Mermin, "Solid State Physics", below. It can be even more subtle in three dimensions!

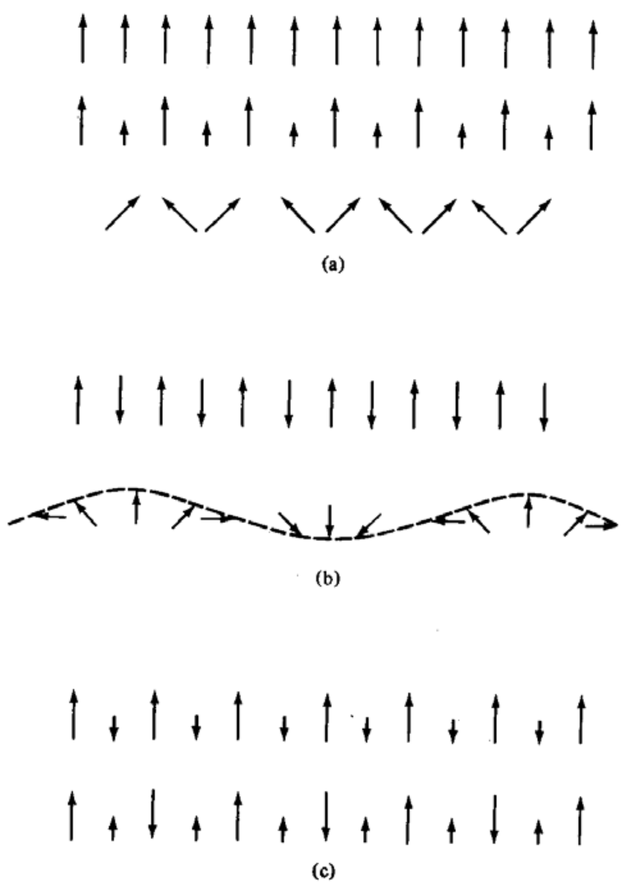
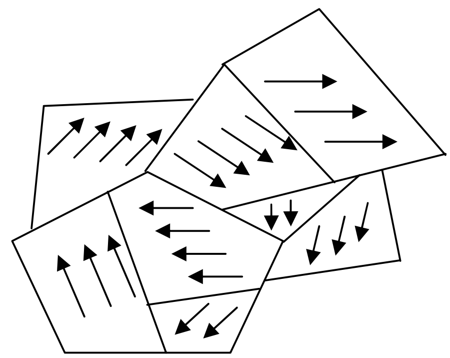


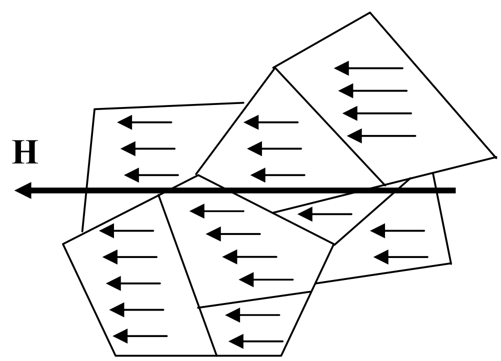
Figure 33.2
Linear arrays of spins illustrating possible (a) ferromagnetic, (b) antiferromagnetic, and (c) ferrimagnetic orderings.

These kinds of ordering are only possible if the thermal excitations in the material are not too big. If thermal vibrations at the atomic level have more energy than the small potential difference due to alignment with a neighbouring atom, then the alignment will not be able to persist. Above some critical temperature it is observed that ferromagnetic, antiferromagnetic and ferrimagnetic materials revert to being simple paramagnetic solids, with a linear susceptibility that obeys the Curie law (2..17). The critical temperature is called the Curie temperature for ferromagnetic (or ferrimagnetic) materials and the Néel temperature for antiferromagnetics.

Let us consider a macroscopic piece of ferromagnetic material as it is cooled through its Curie temperature. Assume first of all that there are no magnetic fields in the region. As the temperature reaches the Curie temperature individual atoms throughout the material begin to form stable alignments with their nearest neighbours due to the short range force. The direction of such an alignment must be arbitrary, since there is no external field to suggest a preferred direction, but the atoms can only go to the lowest energy state by spontaneously breaking the directional symmetry which previously existed. This is the classic model of spontaneous symmetry breaking which occurs in many other physical processes. In a small domain within the material, once a few atoms have spontaneously chosen the direction of their parallel axes, the short range forces with their next neighbours will cause them to line up in the same direction. In many ferromagnetic materials this happens over regions of 10s of micrometres in size.



But inside each of the aligned atoms is an aligned spin, and hence a magnetic moment. The magnetic interactions of these dipoles with their nearest neighbours' dipoles are much weaker than the short range interatomic interactions which cause alignment, but in the volume of a domain there are many millions of atoms, so the magnetic moment of the domain becomes so significant that it generates an appreciable magnetic field upon the whole region around it. It then becomes energetically advantageous for adjacent domains to align themselves in directions which reduce the overall potential energy – in a 3-D version of the way a pair of bar magnets prefer to settle with unlike poles together.



If an external magnetic field **H** is applied

there is a tendency for the domains to become aligned in its direction. Because the moments are so strong the magnetisation of the material can become very large.

Remember (2.11); which can be written as $\mathbf{B}(\mathbf{r}) = \mu_0\left(\mathbf{H}(\mathbf{r}) + \mathbf{M}(\mathbf{r})\right)$. In ferromagnetic materials, B can be much bigger than $\mu_0 H$ because of the large value of M due to the alignment of domains. Ferromagnetism acts as a strong amplifier of magnetic effects.

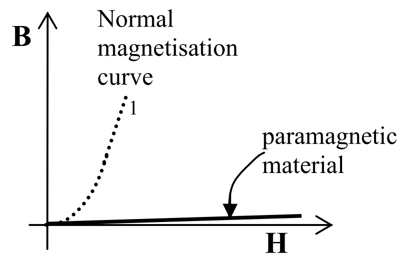
3.2 Nonlinear Properties of Ferromagnetic Material

Since **H** is produced by free currents only – the kind that flow in wires or coils – we can always impose a value of **H** from outside a ferromagnetic sample. The induction **B** depends upon both **H** and the magnetisation **M**. As we change **H**, the magnetisation of the sample will change, and we can detect the combined effect by using Faraday's law to observe the change in **B.** A particular circuit for doing this will be discussed in the next section. We sometimes call **H** the "magnetising force" because it can be imposed on a sample to induce magnetisation.

What is observed as a sample is magnetised; Hysteresis.

Start with an unmagnetised sample below its Curie temperature. All the domains are higgledy-piggledy. There is no external field.

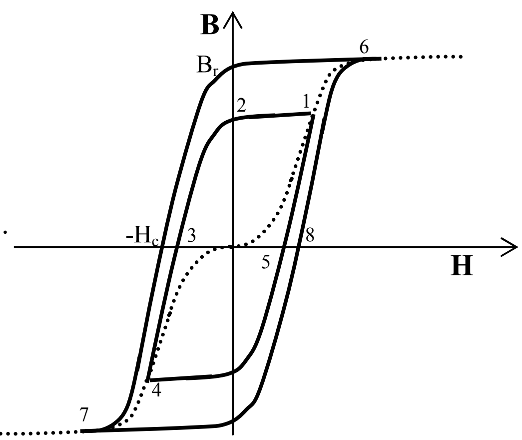
Apply a small magnetising force **H** with a coil wrapped around the sample, carrying a free current *I*. Detect the change of induction **B** in a second coil wrapped around the sample. The sample begins to become magnetised, moving to point 1 ($\mathbf{H}_1$, $\mathbf{B}_1$) on the normal magnetisation curve (dotted line).



Begin to reverse the field **H.** The material "remembers" its magnetisation **M**, so the value of $\mathbf{B}(\mathbf{r})\ (= \mu_0 (\mathbf{H}(\mathbf{r}) + \mathbf{M}(\mathbf{r})))$ does not retrace the normal curve but cuts the $\mathbf{H} = 0$ axis at point 2.

Continue to reverse **H**. The magnetisation begins to drop getting close to zero at point 3, with a finite reverse **H** field. As **H** reaches $-\mathbf{H}_1$ at point 4, **B** goes to $-\mathbf{B}_1$.

Now increase **H** again. The value of B follows the curve through points 4 and 5 back to 1. We have gone once around a "minor hysteresis loop".



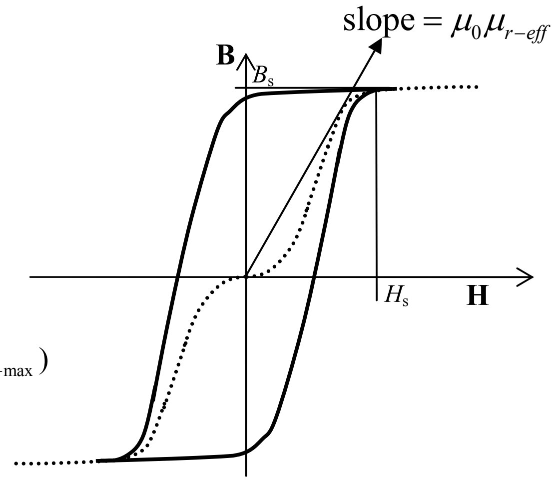
If **H** is increased further from point 1 we follow the normal magnetisation curve until the material cannot be magnetised any more. At point 6 $|\mathbf{M}| \to M_s$, the saturation magnetisation. To the right of point 6 **B** rises very slowly, due only to the $\mu_0 \mathbf{H}$ term. Point 6 has coordinates (H_s, B_s), the saturation values of B and H.

Reversing **H** from anywhere to the right of point 6 takes us around the "major hysteresis loop". The value of |**B**| when **H** reaches zero on this loop is called the "remanence" B_r. The (negative) value of |**H**| required to reduce **B** to zero on this loop is called the "coercivity"Hc. At point 7 the sample is saturated in the reverse direction and the curve from 7 though 8 to 6 completes the major loop.

3.3 Definitions and examples

Remember $\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$ in any material.

Saturation Magnetisation Ms; The value of M when the domains are fully aligned.
Saturation intensity H_s; The magnetic intensity (or magnetising force) required to produce saturation.
Saturation induction B_s; The value of B at the saturation point.
Remanence B_r; The value of B on the major loop when H is brought back to zero from saturation.
Coercivity H_c: The value of H in the reverse direction needed to reduce B to zero after saturation (i.e. on a major loop)
Effective relative permeability μ_{r-eff} $(\equiv \mu_{r-\max})$ the slope/μ_0 of the steepest tangent to the normal magnetisation curve.



Some Ferromagnetic materials

Soft	$\mu_{r\text{-eff}}$	H_c A m^{-1}	B_s (T)
3% Si-Fe	4.0×10^4	8.0	2.0
(Mn-Zn ferrite)	1.5×10^3	0.8	0.2
Mumetal	1.0×10^5	4.0	0.6
Supermalloy	1.0×10^6	0.2	0.8
Hard		H_c A m^{-1}	B_r (T)
5% Chromium steel		5×10^3	0.94
Alnico (high remanence)		8×10^4	0.62
Cobalt-samarium, Co_5Sm		1×10^6	1.50
Fe-Nd-B alloy		1×10^6	1.30

Note the enormous diversity of properties.

"Soft" materials have low coercivity and remanence but very high μ_{r-eff} They are very easy to magnetise and demagnetise so they are used, for instance, in transformer cores which go through many cycles per second or in shielding against external fields.

"Hard" materials have large coercivity and remanence. Once magnetised they are hard to demagnetise so they are used to make permament magnets.

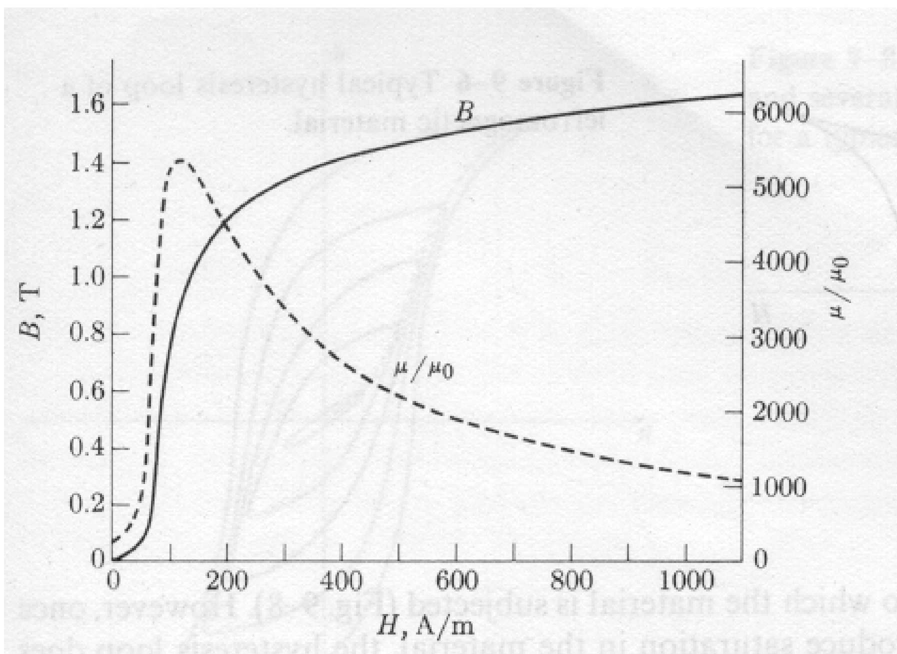
The best electromagnet steels saturate at $B_s \sim 2$ Tesla. This can be (depending on geometry) up to a few thousand times more than the corresponding induction which would be given by the same current through the same magnet coils (i.e. for the same H)

Ferromagnets	Curie Temp (K)	$\mu_0 M_s \simeq B_s$ Tesla
Fe	1043	~2*
Co	1388	~1.6*
Ni	627	~0.6*
Gd	293	1.98
Dy	85	3.0
$GdCl_3$	2.2	0.55

*Depending on annealing, working etc.

Ferrimagnets		
Fe_3O_4 (magnetite)	858	0.51
$CoFe_2O_4$	793	0.475

Antiferromagnets	**Néel Temp (K)**
MnO	122
FeO	198
NiO	600
$MnCl_2$	2

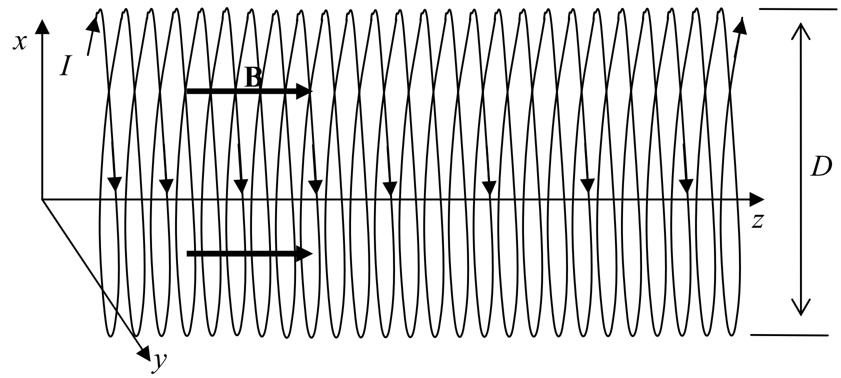


with no ferromagnetic material present. See e.g. graph above for annealed “commercial iron” (from Reitz, Milford and Christie, Addison Wesley)

Some ferrimagnetic materials are also used for permanent magnets. Their largest application is probably for high frequency transformer cores, inductors and radio-aerial cores. Because they are dielectrics they do not dissipate energy by eddy currents (though all materials with hysteresis dissipate some energy every time they go around the loop).

3.3 Straight Cylindrical Magnets

The Solenoid



A solenoid is a tightly packed uniform cylindrical coil carrying current I, with the spacing between turns $d = 1/N$ much smaller than the diameter D of the cylinder on which it is wound. There are N turns per metre of length. First consider the ideal case of a nearly infinite solenoid, length $L>>D$. From symmetry, far from the ends all lines of **B** must be parallel inside the solenoid, i.e. $B_y = 0,\ B_x = 0$. From this we can prove that B_z is the same at any **r** within the coil, so long as there are no currents apart from that in the windings around the outside.

Anywhere inside, from (1.36), $\nabla\times\mathbf{B}(\mathbf{r}) = \mu_0\mathbf{J}(\mathbf{r}) = 0$

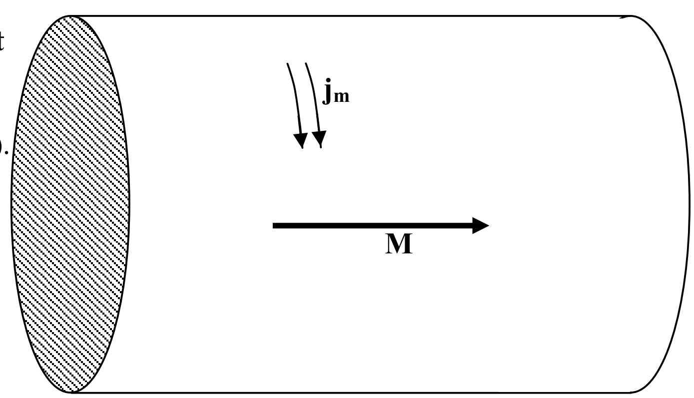
$$= \hat{\mathbf{a}}_x\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) + \hat{\mathbf{a}}_y\left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right) + \hat{\mathbf{a}}_z\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) = 0 .$$

But $B_y = 0,\ B_x = 0$ so their derivatives are also zero,

so
$$\hat{\mathbf{a}}_x\frac{\partial B_z}{\partial y} - \hat{\mathbf{a}}_y\frac{\partial B_z}{\partial x} = 0 .$$

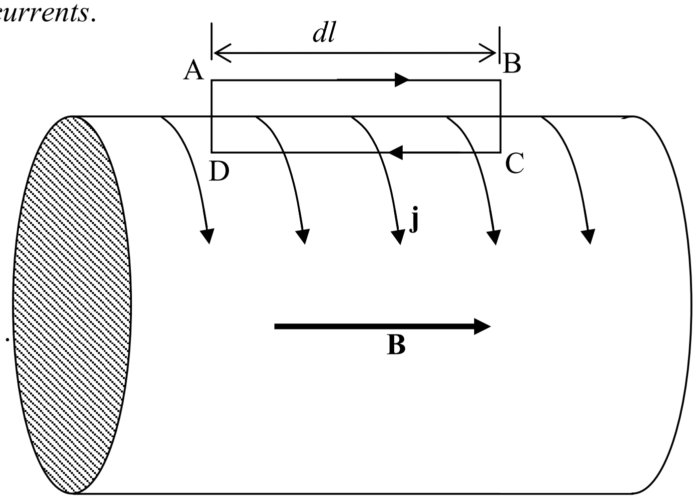
For the vector on the left of this equation to be zero both of its components must separately be zero, that is $\frac{\partial B_z}{\partial y} = 0,\ \frac{\partial B_z}{\partial x} = 0$, so $\mathbf{B} = +\hat{\mathbf{a}}_z B_z$ is uniform everywhere within the solenoid, far from its ends.

Long Cylindrical Bar Magnet
(Assume uniform magnetisation **M** in the +z direction everywhere inside the material). Surface magnetisation current $\mathbf{j_m}$ is perpendicular to **M** (section 2.2) and can be regarded as running around the ouside of the cylinder just like the real current I in the windings of the solenoid. We also saw in section 2.2 that $j_m = |\mathbf{j}_m| = |\mathbf{M}| = M$) **.**



The equivalent surface current density in the windings of the solenoid is in the same direction as for the bar magnet, with $j_f = IN$, that is current/turn × turns/metre. In the solenoid the current I is a free current which is the sole cause of the induction **B**. In the magnet we invented the surface magnetisation current to be the equivalent current that would be needed to cause the induction **B** due to magnetisation **M**.

To get the value of **B** inside either the long solenoid or the long bar magnet we can use the same sort of boundary condition argument we had in section 2.7, except that this time *there are surface currents*.



We do a line integral Around the loop ABCDA Using Ampere's law in its original form (1.33) $\oint_{ABCDA} \mathbf{B}.d\mathbf{l} = \mu_0 I_{loop}$, where I is the current flowing through the loop, $I_{loop} = jdl$. And once again we let the segments BC and DA which cross the boundary become arbitrarily small so that they make no contribution. Then $\oint_{ABCDA} \mathbf{B}.d\mathbf{l} = \mu_0 jdl = \mathbf{B}_{out}.\overrightarrow{AB} + \mathbf{B}.\overrightarrow{CD}$.

If the ends of the magnet or solenoid are a long way from where we are doing the loop integral then the lines of **B** in the space outside will spread out over a dipole pattern into all of the space around on a scale commensurate with the length; so that as the length tends to infinity, B_{out} tends to zero. Thus the only contribution to the integral comes from the term $\mathbf{B}.\overrightarrow{CD} = B_z dl = \mu_0 jdl$, where $j = j_f = NI$ for the long cylindrical solenoid and $j = j_m = M$ for the long cylindrical magnet.

Hence, for the long solenoid $\qquad B_z = |\mathbf{B}| = \mu_0 NI \qquad$ (3.1)

And for the long bar magnet $\qquad B_z = |\mathbf{B}| = \mu_0 M$. $\qquad$ (3.2)

In fact, since **B** and **M** are parallel, $\mathbf{M} = \dfrac{\mathbf{B}}{\mu_0}$.

The two cases have identical patterns of **B**, both inside and outside. The magnetic intensity **H** is very different in the two cases. For the solenoid there is no magnetised material, so $\mathbf{H} = \dfrac{\mathbf{B}}{\mu_0}$ everywhere; inside and out. For the long bar magnet, from (2.11), $\mathbf{H} = \dfrac{\mathbf{B}}{\mu_0} - \mathbf{M}$, but $\mathbf{M} = \dfrac{\mathbf{B}}{\mu_0}$, so H is zero both inside and outside the magnet. We would get the same result if we used the boundary condition from section 2.7 on **H** at the cylindrical surface. There are no real currents there, so the parallel components of **H** must be the same both inside and outside the long magnet. But outside we know that $H_{out} = \frac{B_{out}}{\mu_0} = 0$ since $B_{out} = 0$. Thus **H** must also be zero inside the surface.

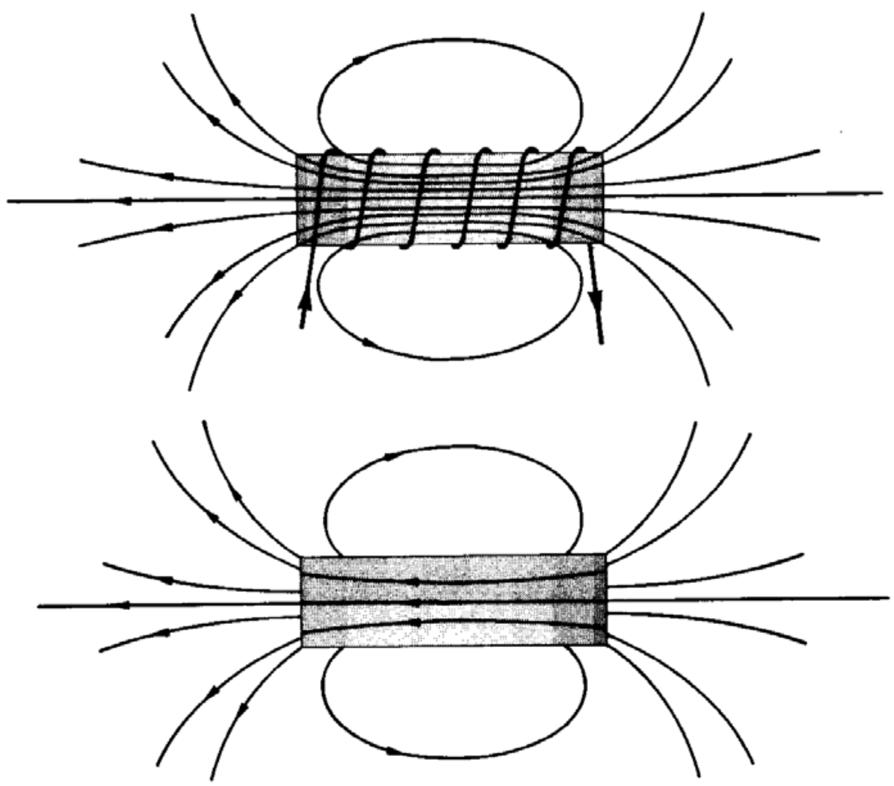
It is straightforward to work out what the fields will be inside a long cylindrical **electromagnet** where a solenoid carrying current *I* is wound around a magnetised cylinder of material with magnetisation **M** pointing along the *z* axis. The the effective surface current flowing around the cylindrical surface becomes $j = j_m + j_f$. The induction remains along the *z*-axis and is the sum of (3.1) and (3.2), i.e.

$$B_z = |\mathbf{B}| = \mu_0 (NI + M) \tag{3.3}$$

In this case the *H* can only depend upon the free currents, i.e. the current in the solenoid windings, so it has the same value as it did in the solenoid

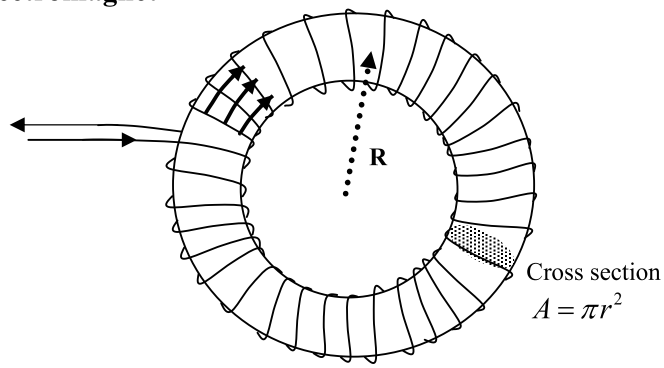
$$H = \frac{B}{\mu_0} = NI \tag{3.4}$$

You will be given a problem in which you have to explain what is happening to **H**, **B** and **M** around a solenoid wound on a magnetised core of *finite* length.



This drawing from Grant and Phillips shows the qualitative behaviour of the **B** field, above, and the **H** field, below, in that case. They point out that, for clarity of illustration, the relative permeability of the material has been set at about 3. (The problem will have a much bigger relative permeability, so the picture for the **H** field will be radically different.)

3.4 Toroidal Electromagnet



A torus is a doughnut shape. If we take a long solenoid wound uniformly on a cylindrical ferromagnetic core and bend it so that the ends meet, then there will be nowhere for the **B** field to get out. Let *N* now represent the total number of turns (not turns per unit length as before). All of the flux of **B** will remain inside the ring . Assume that the minor radius *r* of the torus is much smaller than the major radius *R,* so the material is locally close to straight, and that it has always been magnetised along the local direction of the core at any point. Then we may take **B, H** and **M** as parallel to one another (see arrows) and approximately uniform across the cross section of the torus.

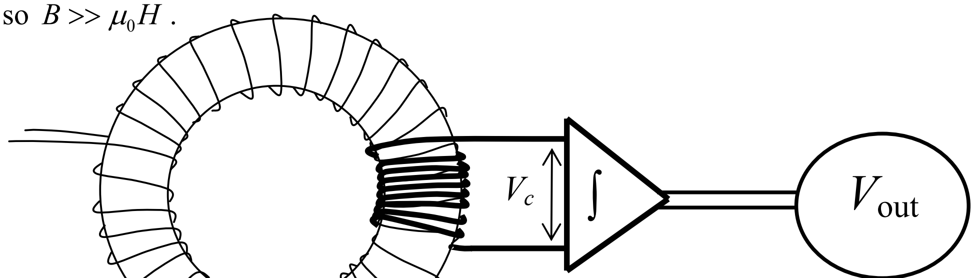
A loop integral at fixed *R* will have the same value of *H* at every point

So $$\oint_{\text{fixed } R} \mathbf{H}.d\mathbf{l} = 2\pi RH = NI$$

And the magnetising force *H* is therefore given by

$$H = \frac{NI}{2\pi R}.$$

We can have complete control over *H,* via *N*, *I* and *R*. But **B** is not directly known because it will also depend upon **M,** since $\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M})$, and **M** depends on the history of magnetisation of the sample. For typical ferromagnetic materials $M >> H$, so $B >> \mu_0 H$.



To determine *B* in a well understood way we wind an extra coil on the toroid, with n_c turns on top of the coil which excites the magnetising force *H*. It is connected to a "fluxmeter", a special electronic circuit represented by the triangle with an integral sign (actually an operational amplifier circuit), which has a high input impedance R_c and the property that its output voltage satisfies the relation

$$V_{out} = K\int_0^t I_c dt \tag{3.5}$$

where $I_c(t)$ is the current flowing into the fluxmeter from the extra coil and K is a calibration constant. [Such a circuit is also called a "charge amplifier", since the integral of current over time t is equal to the charge which flows in that time.]
The current $I_c(t)$ will flow into the impedance R_c because the changing flux **B** in the toroid induces a voltage $V_c(t)$. The flux through a cross section of the core is

$$\Phi(t) = BA = \pi r^2 B(t).$$

Faraday's law in integral form (equation (1.30)) tells us that the voltage around one turn of the extra coil due to changing flux is

$$\Delta V = \oint \mathbf{E}.d\mathbf{l} = -\frac{d\Phi}{dt} = -\pi r^2 \frac{dB}{dt}$$

so around n_c turns $$V_c(t) = n_c\Delta V = \pi r^2 n_c \frac{dB}{dt} = I_c R_c.$$

Putting the value of I_c from this into (3.5) we get

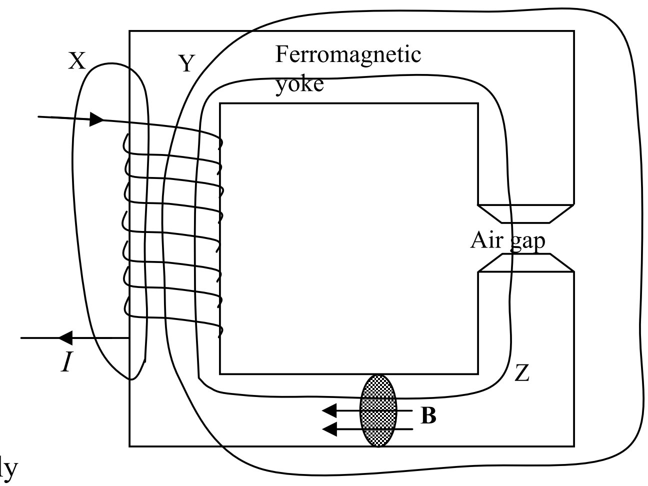
$$V_{out}(t) = \frac{n_c \pi r^2 K}{R_c}\int_0^t \frac{dB}{dt}dt.$$

But the integrand is a perfect differential, $\int_0^t \frac{dB}{dt}dt = \Delta B$, where ΔB is the change of B from time zero to time t. So the output from the voltmeter is

$$V_{out}(t) = \left(\frac{n_c \pi r^2 K}{R_c}\right)\Delta B \tag{3.6}$$

The quantities in brackets are all measurable constants, so the value of V_{out} at time t is proportional to the change of B in the core of the torus since time t. We can use this apparatus to perform the sequence of operations described above in section 3.2, starting with an unmagnetised sample, changing H by changing I in the main coil, and observing the changes in B by reading off the output voltage from a calibrated fluxmeter. It is even possible to connect the ammeter to measure I and the voltmeter to measure V_{out} to the horizontal and vertical deflection plates of a oscilloscope so that hysteresis loops are shown on the screen in real time. This is one of the most direct illustrations of the independent existence of **H** and **B** fields.

3.5 Electromagnets with Gaps



The torus was an ideal electromagnet. More realistic magnets may not have the same cross section everywhere around the yoke, they may have gaps in the yoke, and the coil of N turns carrying current I may all be wound on one part of the yoke ("lumped coil").

A full analysis of the fields in and around such a magnet can only be done by computer. We sketch here the "first order" approximation for estimating the fields and understanding what is going on.

Any line integral around a loop like X, Y or Z shown which goes through the inside of the winding and comes back around the outside, will obey the Ampere law

$$\int_{loop} \mathbf{H}.d\mathbf{l} = NI_{free} = NI$$

This will be true whatever the state of magnetisation of the yoke since magnetisation does not affect free currents. We can take the zero point of l anywhere on the loop.

It is only interesting to build magnets like this if we want to achieve high **B** fields with relatively modest currents, so we always work with highly magnetisable material in conditions where $M >> H$. In these conditions the induction **B** inside the material of the yoke is completely dominated by **M.** This means that there will ONLY be a significant flux EITHER inside the yoke OR across small gaps which are opened in the yoke. At any position l along loop Z (say), with cross section $A(l)$ in the yoke perpendicular to **B**, the flux $\Phi = BA(l)$. But lines of **B** are conserved, so we can treat the yoke as something like an electric circuit through which the "current" Φ is driven by the "magnetomotive force" NI.

To get simple results we make two gross approximations:

a) Within the "circuit" of the yoke plus small gaps, **B**, **H** and **M** are everywhere parallel to one another.
b) The material is linear with $\mu_r = \mu_{r-eff}$ (or "$\mu_{r-\max}$"see hysteresis discussion above)

Then we can set $B(l) = \mu_0 \mu_r H(l)$ at all points around a loop like Z, going along the lines of **B** around the circuit

and
$$NI = \int_{loop} \mathbf{H}.d\mathbf{l} = \int_{loop} \frac{B(l)}{\mu_r \mu_0}.dl = \int_{loop} \frac{\Phi}{\mu_r \mu_0 A(l)}.dl \tag{3.7}$$

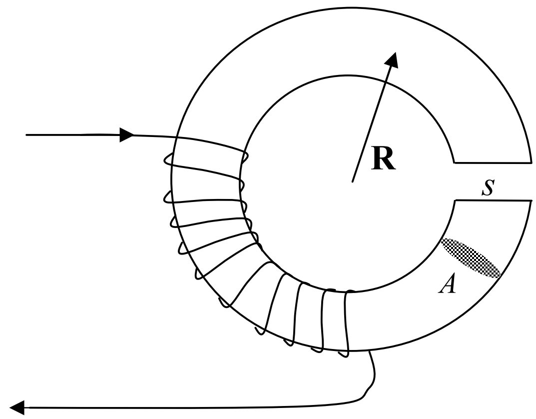
Example. Assume:

i) that the cross section A of the yoke is the same all the way around its length L, that is , not a function of l;

ii) that the width s of the gap in the yoke is very small with flat faces of area A perpendicular to the lines of **B** (simpler that the case drawn above);

iii) the length $\int_{loop} dl \simeq L$ is approximately the same for all loop integration paths parallel to the lines of **B** through the yoke and gap.

This means that our magnet in this simplified case is something like a torus with $L \simeq 2\pi R$ and a lumped coil wrapped on part of it.



Then
$$NI = \int_{loop} \frac{\Phi}{\mu_r \mu_0 A(l)} .dl \simeq \Phi\left(\frac{2\pi R}{\mu_r \mu_0 A} + \frac{s}{\mu_0 A}\right). \tag{3.8}$$

Therefore
$$B \simeq \frac{\Phi}{A} = \frac{NI}{\left(\frac{2\pi R}{\mu_r \mu_0} + \frac{s}{\mu_0}\right)} = \frac{\mu_r \mu_0 NI}{\left(2\pi R + \mu_r s\right)}, \tag{3.9}$$

in yoke **and through gap.**

Notes

1. For a given magnetomotive force NI and a given toroidal yoke, radius R, cross section A, the value of B falls rapidly as the gap is opened up. If the material is ferromagnetic then we expect $\mu_r \approx 1000$, so a 2mm gap in a torus of radius R=30 cm. will approximately halve the value of B compared with the case without a gap.

   To achieve an induction B of 1.5 T in the gap, with s = 2mm, R=30 cm and $\mu_r = 1000$ needs $NI \simeq \dfrac{B}{\mu_r \mu_0}\left(2\pi R + \mu_r s\right) \simeq \dfrac{1.5}{10^3 \times 4\pi 10^{-7}}\left(2+2\right) \simeq 5\times 10^3$ ampere turns,e.g. 5 amps in 1000 turns.

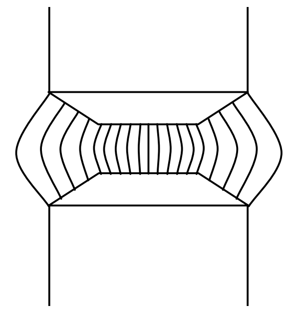
2. Equation (3.8) has the same form as Ohm's law $V = I(R_1 + R_2)$ for two resistances in series. Magnetomotive force NI is analogous to e.m.f. V, flux Φ to current I and the quantities $\dfrac{2\pi R}{\mu_r \mu_0 A}$ and $\dfrac{s}{\mu_0 A}$ to resistances R_{yoke} and R_{gap}. We call these two quantities the "reluctances" of those sections of the magnetic circuit (see textbooks for more).

3. If we want to achieve a higher value of B in the region of the gap than in the yoke itself, then we can use tapered polepieces, as in the magnet sketched at the beginning of this section. Since the flux around the whole magnetic circuit is assumed conserved, then by reducing the cross sectional area A_{gap} compared with area A_{yoke} we can *ideally* magnify the value of $B = \Phi / A$ by a factor A_{yoke} / A_{gap}.

   BUT a) The reluctance of such a gap gets even higher, so more magnetomotive force *NI* is needed to push the flux through it.

   b) Lines of force in free space don't just go across a gap; they actually bulge out, so B_{gap} < ideal.

   c) The poletips that have to carry the higher B field may themselves become saturated. So the maximum value of B is limited to ~ 2 T.



In realistic calculations of magnets, the kind of simple theory given here can be used to understand what is going on and to give starting values, but the approximations are extreme. The only way to get accurate predictions of fields in an actual system is to use a finite-element computer program which splits all of the space in and around the magnet into tiny cells with appropriate magnetic properties, and then solves numerically with the constraints given by the continuity of lines of **B** and the magnetomotive force from the actual turns of the coil.

3.6 Shielding

The idea of reluctance helps us understand what happens if we put a sensitive device like a T.V. tube or a photomultiplier (PMT), whose performance is affected by the earth's magnetic field, inside an enclosure with walls made of soft ferromagnetic material like mumetal or supermalloy (see table in section 3.3 above) with very high μ_r. In the sketch the dotted lines represent the background **B** field without the enclosure. The low reluctance material (shaded) appears to suck-in lines of force from the space around it, reducing the field inside the enclosure.