2B29 Electromagnetic Theory

2. Magnetic Materials

2.1 The Magnetic Dipole
Because there are no magnetic monopoles we have a totally different argument to get the distribution of $\mathbf{B}(\mathbf{r})$ around a magnetic dipole, but the variation with (r, θ, φ) at large distances turns out to be identical.

There are two ways of calculating $\mathbf{B}(\mathbf{r})$ from a current distribution. The analagous way to what we did for the electric dipole would need us to set up something called the magnetic vector potential $\mathbf{A}(\mathbf{r})$ – for which there is not enough time to give a proper treatment in these lectures (though we mention it again – for the same reason as here - when we use an oscillating electric dipole to emit electromagnetic waves: section 14 below). See Grant and Phillips if you are interested. The tougher direct way is to use the Biot-Savart law which you met in 1B26:
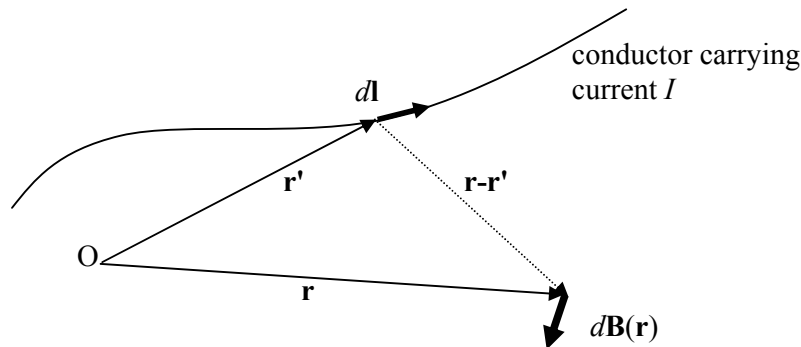
The contribution $d\mathbf{B}(\mathbf{r})$ to the magnetic induction at point $\mathbf{r}$ due to current I through line element $d\mathbf{l}$ at point $\mathbf{r'}$ is, according to the Biot Savart Law

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi} \frac{d\mathbf{l} \times (\mathbf{r} - \mathbf{r'})}{|(\mathbf{r} - \mathbf{r'})|^3} \tag{2.1}$$

The basic magnetic dipole can be a loop of current, shown here as a circle of radius $r' = a$ in the yz plane, centred on the origin.
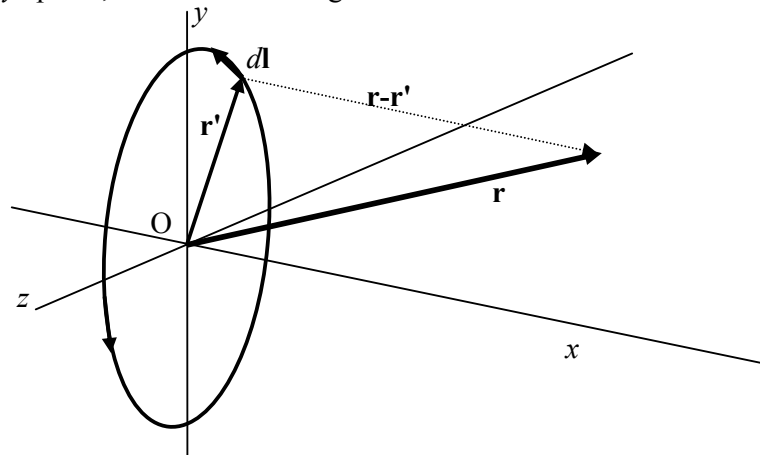
The total field $\mathbf{B}(\mathbf{r}) = \sum_{all\ d\mathbf{l}} d\mathbf{B}(\mathbf{r})$, which in this case is an integral around the current loop. Because of the vector product in (2.1) it is very tricky to visualise what is going on when we do this integral for the general position $\mathbf{r}$. Instead we restrict ourselves to calculating the field for a point on the x axis. The full derivation for any $\mathbf{r}$ can be found in textbooks.
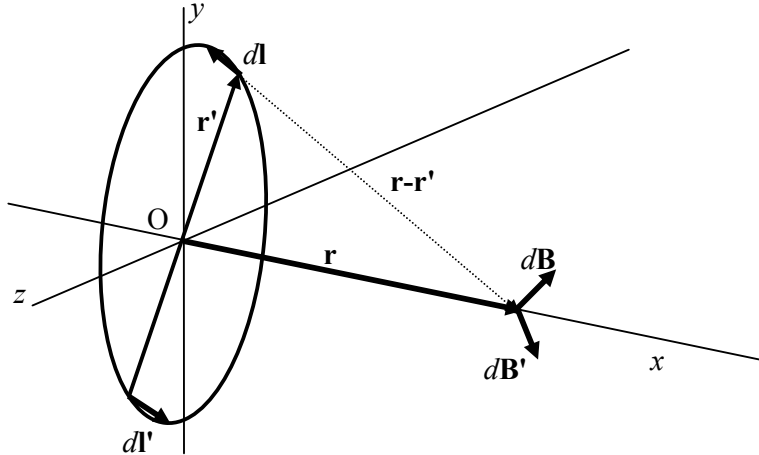

For any $d\mathbf{B}$ due to $d\mathbf{l}$ at a particular $\mathbf{r}'$ there will also be a $d\mathbf{B}'$ at $\mathbf{r}$ due to $d\mathbf{l}'$ at $-\mathbf{r}'$, that is, at the other end of the same diameter of the loop. The components of $d\mathbf{B}$ and $d\mathbf{B}'$ along the x direction will add, but the components perpendicular to x, in the yz plane, will cancel.

In the simple case, where $d\mathbf{l}$ is at $y = a$, $z = 0$; i.e. $\mathbf{r}' = a\hat{\mathbf{j}}$;

then $|\mathbf{r} - \mathbf{r}'| = \sqrt{x^2 + a^2}$ and the x component of $d\mathbf{l} \times (\mathbf{r} - \mathbf{r}')$ is, from standard vector

product rules, $\qquad \hat{\mathbf{i}}(dl_y(\mathbf{r} - \mathbf{r}')_z - dl_z(\mathbf{r} - \mathbf{r}')_y) = +\hat{\mathbf{i}}dla$ \hfill (2.2)

.
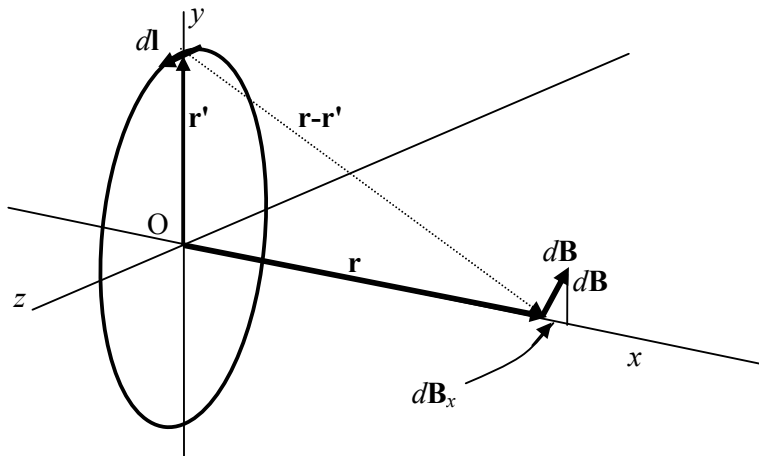
[At this stage of the argument it helps to use the "right hand rule" to make sure you agree with the direction of $d\mathbf{B}$.]

The non-cancelling contribution to $\mathbf{B(r)}$ from this $d\mathbf{l}$ (and every other $d\mathbf{l}$ around the loop) is therefore, from (2.2) in (2.1),

$$d\mathbf{B}_x = \hat{\mathbf{i}}\,\frac{\mu_0 I}{4\pi}\,\frac{dl\,a}{\left(x^2+a^2\right)^{3/2}}. \tag{2.3}$$

This has to be integrated to get the contributions from all the $d\mathbf{l}$ around the loop, the whole circumference, giving a factor $2\pi a$, so

$$B_x = \sum_{loop} dB_x = \frac{\mu_0 I}{4\pi}\,\frac{2\pi a^2}{\left(x^2+a^2\right)^{3/2}} = \frac{\mu_0 I a^2}{2\left(x^2+a^2\right)^{3/2}}.$$

The vector area of the loop is $\mathbf{S} = \hat{\mathbf{i}}\pi a^2$, where we take the positive normal to the surface as pointing along the direction of advance of a right-hand screw thread when rotated in the direction of the current; i.e. advance along the x axis. So we can define the magnetic dipole strength as $\mathbf{m} \equiv I\mathbf{S}$. $\qquad$ (2.4)

Hence $B_x = \dfrac{\mu_0 m}{2\pi\left(x^2+a^2\right)^{3/2}}$. And when $x \gg a$ $\left(x^2+a^2\right)^{3/2} \simeq x^3$.

so, $\qquad\qquad\qquad\qquad B_x \simeq \dfrac{\mu_0 m}{2\pi x^3} \simeq B_r \qquad\qquad\qquad$ (2.5)

for position $\mathbf{r}$ along the x axis at a large distance compared with the size of the current loop. This can be compared with the equivalent $\mathbf{E}$ field due to an electric dipole, see

(1.42) $E_r \simeq \dfrac{m\cos\theta}{2\pi\varepsilon_0 r^3}$, $\quad = \dfrac{m}{2\pi\varepsilon_0 x^3} = E_x$ when we go on axis at large distance compared

with the spacing of charges. The only difference between the electric and magnetic cases is a factor μ_0/ε_0. Both $\mathbf{B}$ and $\mathbf{E}$ are proportional to m and fall off like $1/x^3$.

We could go on to prove from the Biot-Savart law that, at large distances for any position $\mathbf{r}$, all components of $\mathbf{B(r)}$ for the magnetic dipole are the same as those for $\mathbf{E(r)}$ from the electric dipole multiplied by μ_0/ε_0; i.e. following (1.41), (1.42), (1.43)

$$B_\varphi = 0,\; B_r \simeq \frac{\mu_0 m\cos\theta}{2\pi r^3},\; B_\theta = \frac{\mu_0 m\sin\theta}{4\pi r^3} \tag{2.6}$$

What we call the "far field" pattern of lines of force is the same for magnetic or electric dipoles. Close to the dipoles, in the "near field" the pattern is different because one case is based on a pair of charges displaced along the vector $\mathbf{m}$ and the other is based on a current loop in the plane perpendicular to $\mathbf{m}$.
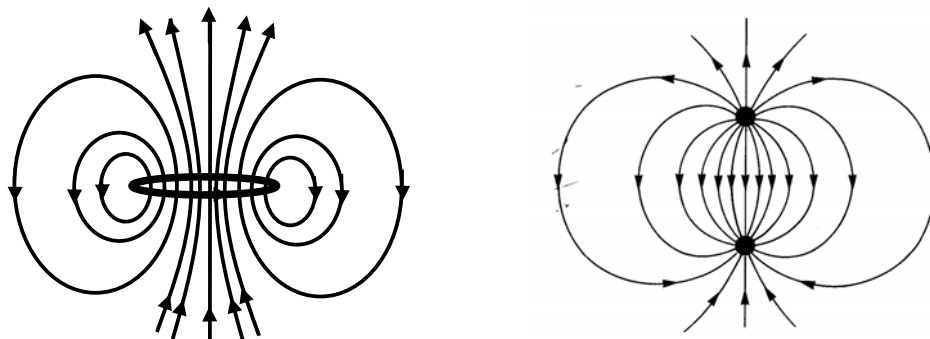
2.2 Uniformly magnetised cylindrical slab; surface magnetisation current.

We assumed in section 1.3 above that an electric field **E** induces in linear dielectric material an electric polarisation **P**, proportional to **E**, with units of dipole moment per unit volume. Then we interpreted this in terms of polarisation charge density ρ_P which is fixed within the material.

In an analogous way we define the magnetisation **M** of a linear magnetic medium. It is the magnetic dipole moment per unit volume, and will be proportional to the applied magnetic induction **B**. Since there are no magnetic monopoles, **M** must be regarded as due to the presence of lots of tiny magnetic dipoles at the atomic or molecular level. Classically these can be thought of as tiny loops of current. At the most fundamental level, in quantum mechanics, these dipoles may actually be associated with the magnetic moments of molecules, atoms or even individual electrons.
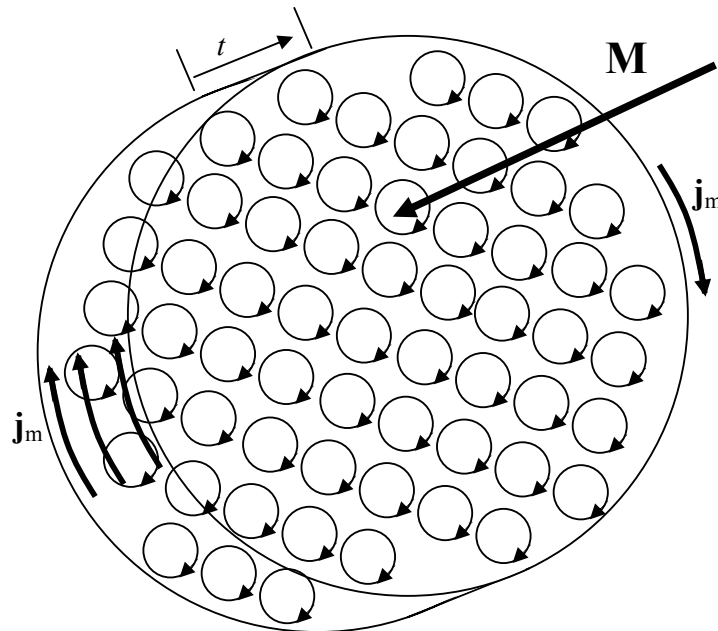
Staying with the classical picture, we can imagine the interior of a cylindrical slab of material, area S, thickness t, volume $V = St$, with uniform magnetisation **M,** as containing a high density of tiny current loops, all packed in side by side. The direction of **M** is normal to the face of the slab.

The effective B field due to these little loops is the same as that of a dipole magnet with magnetic moment $V\mathbf{M} = St\mathbf{M}$. But if we look closely at a small strip ABCD inside the material (or WXYZ), for each current going from up to down there will be a nearby equal and opposite current going from down to up, and for each current going from left to right there will be a nearby current going from right to left.
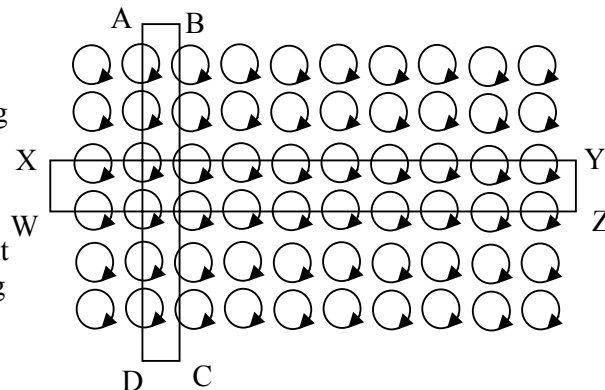
Sensing the field due to these currents from a distance, the internal currents will all cancel out their near neighbours (we could use the Biot Savart law to show that). The only places where there are uncancelled currents are around the boundary where the currents in the outermost loops are unmatched by external neighbours. So there is an effective surface density $\mathbf{j}_m$ of "magnetisation current" or "amperian current" which represents the effect of the magnetisation. This is analogous to the surface polarisation charge density on the faces of the dielectric inside the capacitor in section 1.3 above. We can calculate the size of this effective surface current density because we know that its dipole moment is $St\mathbf{M}$. The current must be flowing around the outside of the cylindrical surface of thickness t. Surface current density $\mathbf{j}_m(\mathbf{r})$ has the dimensions of current/(unit length), so the total current flowing around the slab is $I = j_m t$. From (2.4) therefore $StM = IS$, so $j_m = I / t = M$. The vector $\mathbf{j}_m$ is everywhere perpendicular to $\mathbf{M}$.

2.3 Non-uniformly magnetised Material; Volume Magnetisation Current.

In general $\mathbf{M}(\mathbf{r})$ can vary from place to place inside a magnetised material, as can $\mathbf{B}(\mathbf{r})$. Assume that the value of $\mathbf{M}$ falls off from left to right, so the density of the tiny dipoles is reducing as we move to the right. Then in any strip ABCD there will be more currents going down on the left of the strip than coming up on the right of the strip. So there will be a net magnetisation current $\mathbf{J}_m$ flowing downward. (We use lower case $\mathbf{j}_m$ for surface current density and upper case $\mathbf{J}_m$ for volume current density. Look out; Grant and Phillips are different.)
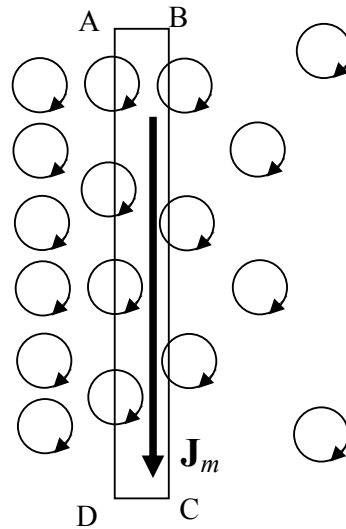
To make this discussion more rigorous, consider the general case where $\mathbf{M}$ has an arbitrary direction in a Cartesian system. Imagine tiny rectangular blocks of material with edges lined up along the *x, y* and *z* axes. Their volumes are $V = \delta x \delta y \delta z$. The first, at position x has component $M_z(x)$ of magnetisation in the z direction. This can be regarded as due to surface magnetisation currents $\mathbf{j}_m$ flowing around the four faces which contain the z direction, as sketched, where $j_m = M_z(x)$.
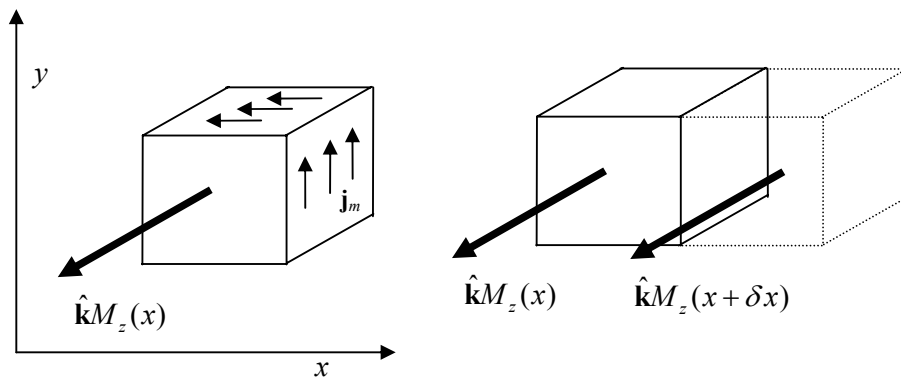
The second block is displaced by δx and has a component $M_z(x+\delta x)$ of magnetisation in the z direction. At their common boundary there will be a surface current density in the $+y$ direction equal to

$j_m(x) - j_m(x+\delta x) = M_z(x) - M_z(x+\delta x) = -\frac{\partial M_z}{\partial x}\delta x$. There are $1/\delta x$ such

boundaries per unit length in the x direction, so the contribution from this term to the

y component J_{my} of the volume current density is $-\frac{\partial M_z}{\partial x}$.

Now look at the effect of the variation of M_x when we go from the first block to a third block sitting in front of the first block, displaced by $+\delta z$. In this case there will be a surface current density in the $+y$ direction on the boundary between the first and third blocks equal to

$$j_m(z+\delta z) - j_m(z) = M_x(z+\delta z) - M_x(z) = \frac{\partial M_x}{\partial z}\delta z$$

This contributes $\frac{\partial M_x}{\partial z}$ to J_{my}. There are only two contributions to J_{my}, so we get

$$J_{my} = \left(\frac{\partial M_x}{\partial z} - \frac{\partial M_z}{\partial x}\right);$$

familiar as part of the standard expression for $curl\mathbf{M}$. Doing a similar exercise with the appropriate pairs of elementary blocks for the variations of $M_x(y)$, $M_y(x)$, $M_z(y)$ and $M_y(z)$, we finally get a complete vector expression for the volume magnetisation current density

$$\mathbf{J}_m(\mathbf{r}) = \hat{\mathbf{i}}\left(\frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}\right) + \hat{\mathbf{j}}\left(\frac{\partial M_x}{\partial z} - \frac{\partial M_z}{\partial x}\right) + \hat{\mathbf{k}}\left(\frac{\partial M_y}{\partial x} - \frac{\partial M_x}{\partial y}\right) = \nabla \times \mathbf{M} \quad (2.7)$$

This expression relating magnetisation current density $\mathbf{J}_m$ to the variation of magnetisation $\mathbf{M}$ is the analogue in magnetic materials to expression (1.17) $\rho_p = -\nabla\mathbf{P}$ which relates polarisation charge ρ_p to the variation of the polarisation $\mathbf{P}$ for electrical materials.

2.4 The Magnetic Intensity H(r)
The argument continues in parallel with that for electrical materials – *but because the definitions are subtly different for electrical and magnetic quantities the formulae are never obvious.*

In (1.36) we had $\nabla \times \mathbf{B} = \mu_0\mathbf{J}$, where the total current density in the medium is

$$\mathbf{J}(\mathbf{r}) = \mathbf{J}_m(\mathbf{r}) + \mathbf{J}_f(\mathbf{r}) \quad (2.8)$$

and the free current $\mathbf{J}_f$ corresponds to the motion of free charges like electrons or holes in a conductor, or charged particles in a vacuum. The magnetisation current $\mathbf{J}_m$ (also called the Amperian current) bears some relation to the currents which might be
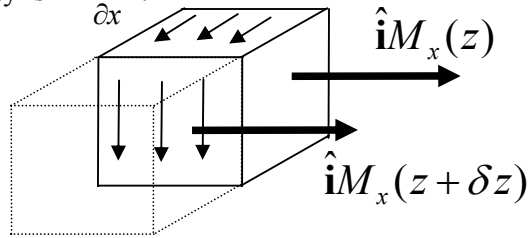
flowing at the molecular or atomic level, but it is primarily introduced as a construct to help describe the effects of magnetisation.

We can rewrite (1.36) as

$$\nabla \times \mathbf{B} = \mu_0 (\mathbf{J}_f + \mathbf{J}_m) \tag{2.9}$$

and substituting from (2.7) this can be rewritten $\nabla \times \left(\dfrac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) = \mathbf{J}_f$. (2.10)

We now define

$$\mathbf{H}(\mathbf{r}) \equiv \left(\frac{\mathbf{B}(\mathbf{r})}{\mu_0} - \mathbf{M}(\mathbf{r}) \right) \tag{2.11}$$

So (2.10) becomes

$$\nabla \times \mathbf{H} = \mathbf{J}_f. \tag{2.12}$$

We can put this back into Stokes' theorem (from Tools) to get

$$\oint_P \mathbf{H}.d\mathbf{l} = \int_{S_P} (\nabla \times \mathbf{H}).d\mathbf{S} = \int_{S_P} \mathbf{J}_f.d\mathbf{S}, \tag{2.13}$$

or if the total free current flowing through area S_p is I_f

$$\oint_P \mathbf{H}.d\mathbf{l} = I_f. \tag{2.14}$$

We call $\mathbf{H}(\mathbf{r})$ the magnetic intensity. (What are its units?) It depends directly upon the influence of free currents, whereas $\mathbf{B}(\mathbf{r})$ depends on both free and magnetisation currents; a close parallel to the dependence of $\mathbf{D}(\mathbf{r})$.directly on free charge, while $\mathbf{E}(\mathbf{r})$ depends on both free and polarisation charge. [Note, in particular situations $\mathbf{H}$ can be without free currents present, and $\mathbf{D}$ nonzero without free charge, even though (1.21), (2.12) and (2.14) are satisfied. You will have some magnetic problems on the sheets where this happens.]

Equations (2.12), (2.13) and (2.14) are all versions of Ampere's law. We are not yet ready to make them part of our set of Maxwell's equations because there is still something missing; see Section 5, below.

In general $\mathbf{H}, \mathbf{B}$ and $\mathbf{M}$ do not have to be parallel to each other. In a ferromagnetic material they are often not parallel to one another – see later in these notes – and their sizes are not necessarily proportional to one another.

We can define a magnetic susceptibility χ_m via $\quad \mathbf{M} = \chi_m \mathbf{H}$. (2.15)

In linear isotropic materials (not in ferromagnetics) χ_m is a constant of the material.

Then

$$\mathbf{H} \equiv \left(\frac{\mathbf{B}}{\mu_0} - \chi_m \mathbf{H} \right),$$

or

$$\mathbf{B} = \mu_0 (1 + \chi_m)\mathbf{H} = \mu_0 \mu_r \mathbf{H} = \mu \mathbf{H}. \tag{2.16}$$

Here μ is the permeability and μ_r is the relative permeability. (Look out, *every* book defines the symbols χ_m, μ and μ_r slightly differently, though they all agree on the concepts.)

2.5 Diamagnetic and Paramagnetic Materials

Susceptibilities of some linear materials at 293K

MATERIAL	SUSCEPTIBILITY χ_m
Gold	-3.6×10^{-5}
Copper	-0.98×10^{-5}
Germanium	-1.5×10^{-5}
Tungsten	$+6.8 \times 10^{-5}$
Aluminium	$+2.3 \times 10^{-5}$
Magnesium	$+1.2 \times 10^{-5}$
Glass	-1.1×10^{-4}
Fused Quartz	-6.2×10^{-5}
Nitrogen (76 cm Hg pressure)	-6.7×10^{-9}
Oxygen (76 cm Hg pressure)	$+1920.0 \times 10^{-9}$
Helium (76 cm Hg pressure)	-1.05×10^{-9}
Argon (76 cm Hg pressure)	-11.0×10^{-9}
Krypton (76 cm Hg pressure)	-11.1×10^{-9}
Xenon (76 cm Hg pressure)	-24.6×10^{-9}
Water	-9.1×10^{-6}
Sodium Chloride	-1.38×10^{-5}

Note   a) some of these materials have $\chi_m > 0$; some have $\chi_m < 0$.
And   b) χ_m is always very small for these linear materials,
         so $\mu_r = 1 + \chi_m \approx 1$ (we often use this approximation when
         magnetic properties are not important; i.e. $\mu = \mu_0 \mu_r \approx \mu_0$)

When $\chi_m > 0$ the material is **paramagnetic**. **M** is parallel to **H**.
When $\chi_m < 0$ it is **diamagnetic**. **M** is antiparallel to **H**.
In both paramagnetic and diamagnetic linear homogeneous materials equations (2.15)
and (2.16) apply; i.e. $\mathbf{M} = \chi_m \mathbf{H}$ and $\mathbf{B} = \mu_0(1 + \chi_m)\mathbf{H} = \mu_0 \mu_r \mathbf{H} = \mu \mathbf{H}$.

Crude explanation of diamagnetism at the atomic/molecular level:

- If there are no intrinsic free magnetic moments (e.g. all atoms have closed shells so no unpaired electrons) then we can consider the electron orbits semi classically; each is like a tiny current loop.
- With no **H** field they are all unexcited so there is no net magnetisation **M**.
- As **H** increases **B** also increases and more flux threads through each loop.
- By Faraday's law this induces a voltage around the loop which tends to produce a current in the loop that opposes the change of flux. That is, an opposing magnetic moment is produced; $\chi_m < 0$, $\mathbf{B} < \mu_0 \mathbf{H}$.

- This mechanism is purely electromagnetic so diamagnetism does not depend upon temperature.

Crude explanation of paramagnetism at the atomic/molecular level:

- If there are free, intrinsic, magnetic moments in the atomic structure of the material (e.g. unpaired electrons), when no external field is applied they will all point in random directions, so **M** will be zero.
- As **H** is increased the dipoles feel a twisting force which tends to align them along the direction of **H**, so $\chi_m>0$, $\mathbf{B} > \mu_0\mathbf{H}$.
- This magnetic ordering is opposed by the random thermal fluctuations in the material. Doing the thermodynamics of this (see textbooks) gives Curie's law

$$\chi_m = \frac{Nm_0^2\mu_0}{3kT} \tag{2.17}$$

  where there are N free dipoles of strength m_0 per unit volume. T is the temperature and k is the Boltzmann constant.
- Few materials are entirely paramagnetic. Most materials with paramagnetic properties also have some electron orbitals which behave like tiny current loops and introduce diamagnetism as well. So for many nominally paramagnetic materials it is more useful to assume

$$\chi_m = \frac{Nm_0^2\mu_0}{3kT} + \chi_{dia},$$

  where $\chi_{dia}<0$ is the diamagnetic susceptibility the material would have if there had been no free dipoles.
- At low temperatures the term with $\chi_m>0$ grows and can dominate, so that some materials behave more and more like pure paramagnetics as they get colder.

2.6 Boundary and Continuity Conditions on B and D

From (1.21), in general **D** satisfies $\nabla\cdot\mathbf{D} = \rho_f$, and we can use Gauss' theorem from our tools $\int_V \nabla\cdot\mathbf{A}\,\mathrm{d}\tau = \oint_S \mathbf{A}\cdot\hat{\mathbf{n}}\,\mathrm{d}S = \oint_S \mathbf{A}\cdot\mathrm{d}\mathbf{S}$ to show that $\oint_S \mathbf{D}.d\mathbf{S} = \int_V \rho_f d\tau$ (c.f. 1.7 and the argument that follows it). If we are in a region where there are no free charges, i.e. $\rho_f = 0$, then **D** will obey the more restricted relations;

$$\nabla.\mathbf{D} = 0 \text{ and } \int_S \mathbf{D}.d\mathbf{S} = 0 \tag{2.18}$$

Because there are no monopoles **B** obeys the same relations (1.28) and (1.27)

$$\nabla.\mathbf{B} = 0 \text{ and } \int_S \mathbf{B}.d\mathbf{S} = 0 \tag{2.19}$$

Taking **D** first, let us consider the smooth boundary surface between two linear dielectric media with no free charges at the surface. Draw a tiny closed "pill box" which has finite flat disc surfaces, area dS, above and below the surface and very short cylindrical walls connecting the two discs.
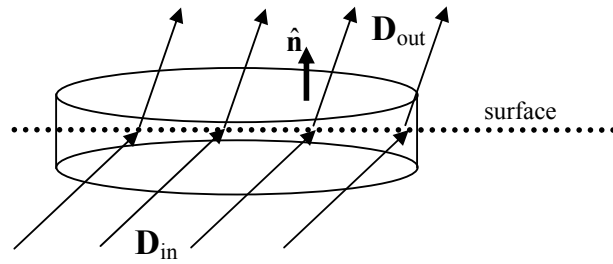
Let both flat surfaces be parallel to each other with vector areas $d\mathbf{S}_{out} = \hat{\mathbf{n}}dS$ above the surface and $d\mathbf{S}_{in} = -\hat{\mathbf{n}}dS$ below it..

Since $\oint_S \mathbf{D}.d\mathbf{S} = \int_V \rho_f d\tau = 0$ in the absence of free charges, the integral of the flux of **D** out of the surface of the box is zero. If we let the cylindrical walls be arbitrarily short, then there is no significant flux through them, so the net flux out of the box is

$$\oint_S \mathbf{D}.d\mathbf{S} = \mathbf{D}_{out}.d\mathbf{S}_{out} + \mathbf{D}_{in}.d\mathbf{S}_{in} = 0$$

Therefore
$$\mathbf{D}_{out}.\hat{\mathbf{n}}dS - \mathbf{D}_{in}.\hat{\mathbf{n}}dS = 0$$

So
$$\mathbf{D}_{out}.\hat{\mathbf{n}} = \mathbf{D}_{in}.\hat{\mathbf{n}} \tag{2.20}$$

That is, the component of **D** perpendicular to the surface has the same value on both sides. This also means that the number of lines of **D** coming into the surface on one side through area dS is equal to the number leaving from the same area on the other side; i.e. lines of **D** are conserved <u>at a boundary with no free charges</u>.

Since the expressions (2.19) for **B** are identical in form to those (2.18) for **D** we can use the identical argument with the pillbox to show that
$$\mathbf{B}_{out}.\hat{\mathbf{n}} = \mathbf{B}_{in}.\hat{\mathbf{n}} \tag{2.21}$$

at a surface between two magnetic materials – since there can be no monopoles on the boundary. That is, the component of **B** perpendicular to the surface has the same value on both sides and lines of **B** are <u>always </u>conserved.

2.7 Boundary and Continuity Conditions on H and E

From (1.30) and (1.32), **E** satisfies $\oint_C \mathbf{E}.d\mathbf{l} = -\dfrac{d\Phi_C}{dt}$ and $\nabla \times \mathbf{E} = -\dfrac{d\mathbf{B}}{dt}$ (2.22)

while from (2.12) and (2.14) **H** satisfies $\nabla \times \mathbf{H} = \mathbf{J}_f$ and $\oint_P \mathbf{H}.d\mathbf{l} = I_f$. (2.23)

In the absence of changing magnetic fields (2.22) becomes
$$\oint_C \mathbf{E}.d\mathbf{l} = 0 \text{ and } \nabla \times \mathbf{E} = 0; \tag{2.24}$$

and in the absence of free currents (2.23) becomes
$$\oint_P \mathbf{H}.d\mathbf{l} = 0 \text{ and } \nabla \times \mathbf{H} = 0 \tag{2.25}$$

(Remember, C was a closed circuit, P was a closed path. They are equivalent)
To get boundary conditions for **E** and **H** we consider a closed path integral ABCD which has vector steps $\overrightarrow{AB} = -\overrightarrow{CD}$ and $\overrightarrow{BC} = -\overrightarrow{DA}$. Now let the steps *BC* and *DA,* which cut the surface, become arbitrarily small so that they make a negligible contribution to $\oint_{ABCD} \mathbf{E}.d\mathbf{l}$.
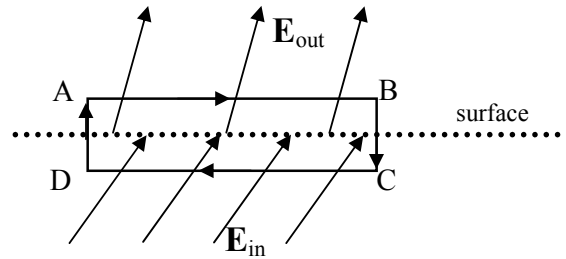
But, from (2.24)

$$\oint_{ABCD} \mathbf{E}.d\mathbf{l} = \mathbf{E}_{out}.\overrightarrow{AB} + \mathbf{E}_{in}.\overrightarrow{CD} = 0$$

So

$$\mathbf{E}_{out}.\overrightarrow{AB} = -\mathbf{E}_{in}.\overrightarrow{CD} = \mathbf{E}_{in}.\overrightarrow{AB} . \qquad (2.26)$$

That is, the components of **E** parallel to the surface are the same above and below the surface. (Since we shrink the area of the loop integral ABCD to zero when we let BC and DA become arbitrarily short, there will be no contribution to $\oint_{ABCD} \mathbf{E}.d\mathbf{l}$ from changing magnetic flux. We could have got (2.26) from (2.22). We set up (2.24) in order to make a parallel argument for **E** and **H**.). Lines of **E** are not, in general, conserved at the boundaries of dielectric materials.

The argument for **H** follows the same track, starting from (2.25) $\oint_{ABCD} \mathbf{H}.d\mathbf{l} = 0$ and taking the same limit of *BC* and *DA* becoming vanishingly small. We there for get (c.f. (2.26))

$$\mathbf{H}_{out}.\overrightarrow{AB} = -\mathbf{H}_{in}.\overrightarrow{CD} = \mathbf{H}_{in}.\overrightarrow{AB} . \qquad (2.27)$$

That is, the components of **H** parallel to the surface are the same above and below the surface in the absence of free currents flowing in the surface (see step from (2.23) to (2.25)). Amperian or magnetisation currents $\mathbf{j}_m$ do not alter this boundary condition, so it can be applied at the surfaces of magnetised materials. As for **E**, lines of **H** are not, in general conserved at a boundary between two regions of different magnetisation.